**Study Guide #4**

**Vorticity**

1. Write out the vector and tensor forms of vorticity.
   1. 
   2. tensor form:
   3. Is this a scalar, first or second order tensor??
2. Vorticity is twice the “local” rotation rate of the fluid – describe what is happening here and relate it to the mathematical definition.
3. The divergence of the vorticity is zero:
   1.  this can be shown by taking the divergence of the curl of a vector – write this out and see.
   2. “Vortex tubes” can be defined as a small tube wrapped around and tangent to the vorticity vector – draw such a tube in general and describe the velocity filed normal to this line.
4. Vorticity can be generated in a flow – an example is flow over a solid surface.
   1. What physical effect is happening to generate vorticity near the surface?
   2. Does vorticity remain at the surface or can it extend out into the flow not touching the surface?
5. Irrotational flow obviously has zero vorticity everywhere. Does this say something about how the partial derivatives of the velocity components are related to each other? This acts as a constraint on how the fluid motion behaves.
6. Circulation is a measure of vorticity on a plane or specified area in the flow.
   1.  Stokes theorem is used to go from the first to the second expression which is a general identity for any vector V.
   2. As can be imagined the larger the circulation the greater the integrated vorticity over a given area.
   3. The area of integration can be chosen arbitrarily and it can include a surface, or body (like a wing).
   4. Now think back to 4.a & b and relate circulation to vorticity generation.
7. Take Euler’s Eqn. and rewrite it using vorticity.
   1. To do this we need a vector identity: 
   2. The above replaces the convective acceleration term which now has 2 parts, one with vorticy and the other not so much.
   3. Note that V dot V is a scalar with magnitude of velocity squared.
   4. Euler’s eqn. becomes:
      1. 
      2. So you may ask, what the….why do this?? Well it helps in the integration to get the more general Bernoulli’s eqn.
      3. Note that the term on the right is a vector with direction normal to both velocity and vorticity. Sometimes it is nice to “call” this term  where  is some scalar that must satisfy its definition.
      4. Now dot the eqn. in 7.d with dl (that is project each term along the line dl that is arbitrary). And then integrate along l between two points we get the general form of Bernoulli’s Eqn.
         1. 
         2. Note that f(t) is a consequence of integrating along dl where P and h terms could be time dependent.
         3. Also 
8. Restricted forms of Bernoulli’s eqn. can be written.
   1. Steady flow (first term is zero and f(t) is zero or a constant.
   2.  is constant (vorticity and velocity vectors are parallel OR the vorticity is zero).
   3. Incompressible fourth term is P/.
   4. Irrotational flow:  is constant but we do more! But to do this introduce a velocity potential, .
9. Reduce the eqn. for each of the above conditions.
10. Note if integrating along a stream line then the vorticity term goes away because dotting  with dl along a streamline much give zero since the former is normal to a streamline!
11. If irrotational the vorticity term goes away!